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Joel Berman* (jberman@uic.edu), Dept. of Mathematics (m/c 249), University of Illinois at Chicago, 851 S. Morgan, Chicago, IL 60607. *Free algebras and hereditarily closed families*. Preliminary report.

Let K be either the set of all positive integers or some initial segment of them. Suppose $\mathcal{K} = \{\mathbf{A}_k \mid k \in K\}$ is a family of pairwise nonisomorphic algebras, all of the same similarity type, indexed by K . We say \mathcal{K} is *hereditarily closed* if for each $k \in K$ every subalgebra of \mathbf{A}_k is isomorphic to \mathbf{A}_p for some $p \in K$ with $1 \leq p \leq k$. Let $m(k, p)$ denote the number of subalgebras of \mathbf{A}_k that are isomorphic to \mathbf{A}_p .

Suppose \mathcal{V} is any locally finite variety generated by some hereditarily closed family of algebras. We provide an upper bound for the cardinality of the n -generated free algebra for \mathcal{V} based on the values of the $m(k, p)$ and we characterize those varieties for which the upper bound is obtained. We also present several illustrative examples of how this upper bound may be computed and applied. (Received January 26, 2009)