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Let  $A$  be an infinite set of generators for a group  $G$ , and let  $S_A(r)$  denote the set of elements of  $G$  whose word length with respect to  $A$  is exactly  $r$ . There are two cases. In the first case, the set  $S_A(r)$  is infinite for all  $r \geq 1$ . In the second case, there is a positive integer  $r$  such that  $S_A(r')$  is infinite for all  $r' < r$  and  $S_A(r') = \emptyset$  for all  $r' > r$ , and  $S_A(r)$  is nonempty, possibly finite. Let  $s$  denote the number of elements in  $S_A(r)$ . The ordered pair  $(r, s)$  is called the *phase transition* of the group  $G$  with respect to  $A$ , and  $S_A(r)$  is called the *transition set*. Given a group  $G$ , it is an open problem to determine the possible phase transitions and transition sets associated with infinite generating sets for  $G$ . This problem is solved for finite transition sets for the additive group  $\mathbf{Z}$  of integers, and some results are known about infinite transition sets of integers. A classification of phase transitions and transition sets is not available even for the group  $\mathbf{Z} \times (\mathbf{Z}/2\mathbf{Z})$ . (Received October 29, 2008)