

1047-11-12

Melvyn B. Nathanson* (melvyn.nathanson@lehman.cuny.edu), Department of Mathematics, Lehman College (CUNY), Bronx, NY 10468. *Phase transitions in infinitely generated groups, and a problem in additive number theory.*

Let A be an infinite set of generators for a group G , and let $S_A(r)$ denote the set of elements of G whose word length with respect to A is exactly r . There are two cases. In the first case, the set $S_A(r)$ is infinite for all $r \geq 1$. In the second case, there is a positive integer r such that $S_A(r')$ is infinite for all $r' < r$ and $S_A(r') = \emptyset$ for all $r' > r$, and $S_A(r)$ is nonempty, possibly finite. Let s denote the number of elements in $S_A(r)$. The ordered pair (r, s) is called the *phase transition* of the group G with respect to A , and $S_A(r)$ is called the *transition set*. Given a group G , it is an open problem to determine the possible phase transitions and transition sets associated with infinite generating sets for G . This problem is solved for finite transition sets for the additive group \mathbf{Z} of integers, and some results are known about infinite transition sets of integers. A classification of phase transitions and transition sets is not available even for the group $\mathbf{Z} \times (\mathbf{Z}/2\mathbf{Z})$. (Received October 29, 2008)