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Richard J. McIntosh* (mcintosh@math.uregina.ca), Department of Mathematics and Statistics, University of Regina, Regina, Sask. S4S0A2, Canada. *Asymptotics and transformations of some Mordell integrals.*

In his last letter to Hardy, Ramanujan used asymptotic expansions to define mock theta functions. The asymptotic expansion of a mock theta involves the asymptotic expansion of certain Mordell integrals. I will show how to obtain the asymptotic expansion and modular transformation of the Mordell integrals

$$J_c(r, \alpha) = \int_0^\infty e^{-\alpha x^2} \frac{\cosh r\alpha x}{\cosh \alpha x} dx \quad \text{and} \quad J_s(r, \alpha) = \int_0^\infty e^{-\alpha x^2} \frac{\sinh r\alpha x}{\sinh \alpha x} dx.$$

The transformation laws are:

$$\sqrt{\frac{\alpha^3}{\pi^3}} J_c(r, \alpha) = 2 \cos\left(\frac{\pi r}{2}\right) \int_0^\infty e^{-\beta x^2} \frac{\cosh \beta x}{\cos \pi r + \cosh 2\beta x} dx$$

and

$$\sqrt{\frac{\alpha^3}{\pi^3}} J_s(r, \alpha) = \sin(\pi r) \int_0^\infty \frac{e^{-\beta x^2}}{\cos \pi r + \cosh 2\beta x} dx$$

for $|r| < 1$, where $\beta = \pi^2/\alpha$. (Received January 29, 2009)