

1047-11-6

Hung-ping Tsao* (hptsao@hotmail.com), 1151 Highland Drive, Novato, CA 94949. *Powered sum formulas via term-wise integrations: a geometric point of view.*

We shall apply the established method for the natural sequence to more general cases such as $S(2n-1;k)$, the k -powered sum of $1, 2, 4, 5, 7, 8, \dots, 3n-2$ and $S(2n;k)$, the k -powered sum of $1, 2, 4, 5, 7, 8, \dots, 3n-2, 3n-1$, where $S(2n-1;1)=[3,-3,1]$, the quadratic polynomial with coefficients $3,-3,1$ and $S(2n;1)=[3,0,0]$. Let $I(S(2n-1;k))$ and $I(S(2n;k))$ be the integrals of $S(2n-1;k)$ and $S(2n;k)$ with respect to n , respectively. Then we can use mathematical induction to prove that $S(2n-1;2)=6I(S(2n-1;1))+cn+d$ and $S(2n;2)=6I(S(2n;1))+cn$ ($d=0$, in the case that the 1-powered sum is a quadratic polynomial without constant term), where c and d can be determined by taking different values of n . Thus we can obtain $S(2n-1;2)=[6,-9,6+c,d]$, where c and d can be determined by solving $1=6-9+6+c+d$ and $1+4+16=48-36+12+2c+d$ so that $S(2n-1;2)=[6,-9,5,-1]$. Likewise, $S(2n;2)=[6,0,-1,0]$. We can then obtain $S(2n-1;3)=9I(S(2n-1;2))+[c,d]=[13.5,-27,22.5,-9,1]$ and $S(2n;3)=9I(S(2n;2))+[c,0]=[13.5,0,-4.5,0,0]$. The most general sequences that this method can be applied to is when the sum of the first $a+b$ terms of which is a quadratic polynomial, because it is equivalent to term-wise integrations justifiable by the fact that the volume of a k dimensional cube is the integral of its surface area with respect to the side. (Received September 19, 2008)