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*Amoebas and coamoebas, relationships and similarities.*

Let  $V$  be an algebraic hypersurface in  $(\mathbb{C}^*)^n$ . We show that the complement components of the coamoeba of  $V$  in the flat torus  $(S^1)^n$  have similar properties as the complement components of the amoeba of  $V$  in  $\mathbb{R}^n$ . More precisely, if  $co\mathcal{A}_V$  is the coamoeba of  $V$ , then we prove that the connected components of  $(S^1)^n \setminus \overline{co\mathcal{A}_V}$  are convex and their number cannot exceed  $n! \text{Vol}(\Delta)$ , where  $\Delta$  is the Newton polytope of the polynomial defining  $V$ , and  $\overline{co\mathcal{A}_V}$  is the closure of  $co\mathcal{A}_V$  in the real torus  $(S^1)^n$ . In addition, we prove that the area of the coamoeba of a complex algebraic plane curve counted with multiplicity cannot exceed  $2\pi^2 \text{Area}(\Delta)$ , and the equality holds if and only if the curve is a Harnack, possibly with ordinary real isolated double points. In the same way, we show that a polynomial  $f$  defining a Harnack curve is dense i.e., its support is  $\Delta \cap \mathbb{Z}^2$ . Using geometric properties of the coamoebas and the logarithmic Gauss map, we give a second proof of Passare-Rullgård's conjecture, that the amoeba of a complex algebraic hypersurface defined by a maximally sparse polynomial is solid. (Received January 15, 2009)