

1047-30-249

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In his celebrated paper on area distortion under planar quasiconformal maps (Acta 1994), K. Astala proved that a compact set  $E$  of Hausdorff dimension  $d$  is mapped under a  $K$ -quasiconformal map  $f$  to a set  $fE$  of Hausdorff dimension at most  $d' = \frac{2Kd}{2+(K-1)d}$ , and that this result is sharp. He conjectured (Question 4.4) that if the Hausdorff measure  $\mathcal{H}^d(E) = 0$ , then  $\mathcal{H}^{d'}(fE) = 0$ .

This conjecture was known if  $d' = 0$  (obvious),  $d' = 2$  (Ahlfors), and recently  $d' = 1$  (Astala, Clop, Mateu, Orobitg and UT, Duke 2008.) The approach in the last mentioned paper does not generalize to other dimensions.

Astala's conjecture was shown to be sharp (if it was true) in the class of all Hausdorff gauge functions by UT (IMRN, 2008).

Finally, we (the 3 named authors) jointly proved completely Astala's conjecture in all dimensions. The ingredients of the proof come from Astala's original approach, geometric measure theory, and some new weighted norm inequalities for Calderón-Zygmund singular integral operators which cannot be deduced from the classical Muckenhoupt  $A_p$  theory.

These results are intimately related to (not yet fully understood) removability problems for various classes of quasiregular maps.

The talk will be self-contained. (Received January 29, 2009)