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**H.-Q. Bui** and **R. S. Laugesen\*** (Laugesen@illinois.edu), Department of Mathematics,  
University of Illinois, Urbana, IL 61801. *Frequency-scale frames and the solution of the Mexican  
hat problem.* Preliminary report.

We resolve a twenty year old open problem on  $L^p$  completeness of the time-scale (or wavelet) system generated by the Mexican hat function, when  $1 < p < \infty$ .

Our main result concerns frequency-scale systems generated by modulation and dilation of a single function. The mixed frame operator (analysis followed by synthesis) is shown to be bijective from  $L^q$  to itself, for  $1 < q < \infty$ , and also from  $W_*^{1,2}$  to itself, so that the frequency-scale synthesis operator is surjective onto those spaces. Tools include the discrete Calderon condition and a generalization of the Daubechies frame criterion in  $L^2$ .

Completeness of the Mexican hat and other time-scale systems in  $L^p$ ,  $1 < p < \infty$ , then follows by Fourier imbedding of the frequency-scale systems. (Received January 30, 2009)