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Sergey Kitaev, Jeffrey Liese, Jeffrey Remmel and Bruce Sagan* (sagan@math.msu.edu),
Department of Mathematics, Michigan State University, East Lansing, MI 48824. *Rationality,
irrationality, and Wilf equivalence in generalized factor order.*

Let P be a poset and consider the free monoid P^* of all words over P . If $w, w' \in P^*$ then w' is a factor of w if there are words u, v with $w = uw'v$. Define generalized factor order on P^* by letting $u \leq w$ if there is a factor w' of w having the same length as u such that $u \leq_P w'$, where the comparison of u and w' is done componentwise in P . One obtains ordinary factor order by insisting that $u = w'$.

Given $u \in P^*$, we prove that the language $\{w : w \geq u\}$ is accepted by a finite state automaton. If P is finite then it follows that the generating function $F(u) = \sum_{w \geq u} w$ is rational. This is an analogue of a theorem of Björner and Sagan for generalized subword order.

We also consider the case when P is the positive integers so that P^* is the set of compositions. Then one obtains $F(u; t, x)$ by substituting tx^n each time the integer n appears in $F(u)$. We can show that $F(u; t, x)$ is also rational. Words u, v are said to be Wilf equivalent if $F(u; t, x) = F(v; t, x)$ and we can prove various equivalences combinatorially.

Björner found a formula for the Möbius function of ordinary factor order. Using the Pumping Lemma we show that $M(u) = \sum_{w \geq u} |\mu(u, w)|w$ can be irrational. (Received November 20, 2008)