

1048-53-5

Abraham D Smith* (adsmith@math.duke.edu), Mathematics Department, Duke University, Box 90320, Durham, NC 27708-0320. *Integrability of 2nd order PDE and the geometry of $GL(2, \mathbb{R})$ -structures.*

A $GL(2, R)$ structure on a manifold of dimension $n + 1$ corresponds to a distribution of rational normal cones over the manifold. Such a structure is k -integrable if there exist submanifolds of dimension k whose tangent spaces are spanned by vectors in the cones.

This structure was first studied by Bryant ($n = 3, k = 2$) in the search for exotic holonomies. Recent work by Ferapontov, et al., showed that the integrability of second-order PDE on $u : R^3 \rightarrow R$ by means of hydrodynamic reductions implies 3-integrability of a natural $GL(2)$ -structure over M^5 . ($n = 4, k = 3$). Ferapontov, et al., also showed that there is an open orbit of such PDE.

Using the techniques of Cartan, we study the equivalence and k -integrability of $GL(2)$ -structures for arbitrary n and k . For $n = 4, k = 3$, we discover a complete classification of local integrable structures into 54 orbits, by the action of $GL(2)$ on binary octic polynomials. This allows explicit construction of all second-order PDE which are integral by hydrodynamic reductions.

Also, the interesting geometry is essentially restricted to this case, as increasing n or k forces the the $GL(2)$ -structure to be flat.

This work is from my PhD thesis, completed March 2009, directed by Robert Bryant at Duke. (Received January 31, 2009)