

1048-57-1

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Surfaces in finite covers of 3-manifolds: The virtual Haken conjecture.

As with many areas of topology and geometry, a starting point in the study of 3-manifolds is to try to understand codimension one objects in them, namely embedded surfaces. A particularly useful class of surfaces are the “incompressible” ones which are topologically essential; a 3-manifold containing such a surface is called a Haken manifold. There are many 3-manifolds which are not Haken, but if we ask about immersed, rather than embedded, surfaces the situation becomes much more mysterious. A closely related question is this: Suppose M is a 3-manifold with infinite fundamental group, does M have a finite cover which is Haken? The Virtual Haken Conjecture posits that the answer to this question is yes.

This talk will survey some recent results in this area, focusing on my work with (variously) William Thurston, Dylan Thurston, and Frank Calegari. From the point of view of Thurston’s Geometrization Conjecture, this is really a question about hyperbolic 3-manifolds, that is, lattices in $\mathrm{PSL}(2, \mathbb{C})$. This opens the door to a rich array of tools that might seem quite surprising in light of the purely topological description of the problem above. Indeed, some unlikely-sounding terms that I will probably mention in my talk are “the Classification of Finite Simple Groups” and “the Langlands Conjecture”, as well as such topological oddities as “random 3-manifolds”! (Received April 10, 2008)