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Bounding relative entropy between measures on product spaces

by the relative entropy between local specifications.

Let $q^n(x^n)$ be the density function of a random vector X^n in R^n . Under the conditions that its local specifications satisfy transportation-cost inequalities, and the dependence of coordinates is weak, we prove a bound for relative entropy in terms of single phase conditional entropies:

$$\begin{aligned} & Ent(p^n || q^n) \\ & \leq \text{const.} \cdot \sum_{i=1}^n Ent(P_i(\cdot | Y_1, Y_2, \dots, Y_{i-1}, Y_{i+1}, \dots, Y^n) || Q_i(\cdot | Y_1, Y_2, \dots, Y_{i-1}, Y_{i+1}, \dots, Y^n)). \end{aligned}$$

(Here Y^n is a random vector with density p^n , and P_i and Q_i are the local specifications of p^n resp. q^n .)

Since single phase conditional entropies measure how different the local specifications of p^n and q^n are, this inequality allows to conclude from closeness of local specifications to closeness of p^n and q^n . Moreover, it yields logarithmic Sobolev inequalities in R^n , under conditions, similar to, but weaker than, those in a recent paper by Otto and Reznikoff. It gives a worse constant, but of the same order of magnitude as by Otto and Reznikoff.

The proof exploits and demonstrates the close connection between entropy and quadratic Wasserstein distance. (Received February 03, 2009)