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**Aleksandar Jurisic\*** (aj@fri.uni-lj.si), Faculty of Computer and Information Science, LKRV, Jadranska 21, 1000 Ljubljana, Slovenia, and **Paul Terwilliger** (terwilli@math.wisc.edu), Department of Mathematics, Van Vleck Hall, University of Wisconsin-Madison, 480 Lincoln Drive, Madison, WI 53706. *Distance-regular graphs with tails*. Preliminary report.

Let  $\Gamma$  be a distance-regular graph with valency  $k \geq 3$  and diameter  $d \geq 2$ . It is well-known that the Schur product  $E \circ F$  of any two minimal idempotents of  $\Gamma$  is a linear combination of minimal idempotents of  $\Gamma$ . Situations where there is a small number of minimal idempotents in the above linear combination can be very interesting. In the case when  $E = F$ , the rank one minimal idempotent  $E_0$  is always present in this linear combination and can be the only one only if  $E = E_0$  or  $E = E_d$  and  $\Gamma$  is bipartite. We study the case when  $E \circ E \in \text{span}\{E_0, E, H\}$  for some minimal idempotent  $H$  of  $\Gamma$ . We call a minimal idempotent  $E$  with this property a *tail*. If  $\Gamma$  is Q-polynomial wrt  $E$ , then  $E$  is a tail. Let  $\theta$  be an eigenvalue of  $\Gamma$  with multiplicity  $m > 1$ . We show that

$$m(a_1 - k - k\omega) \left( \omega - \frac{k\omega^2 - a_1\omega - 1}{k - a_1 - 1} \right) \leq k(a_1^* - m - m\omega) \left( \omega - \frac{m\omega^2 - a_1^*\omega - 1}{m - a_1^* - 1} \right),$$

where  $\omega = \theta/k$  and  $a_1^* = q_{ii}^i$  if  $\theta = \theta_i$ . Let  $E$  be the minimal idempotent corresponding to  $\theta$ . The equality case is equivalent to  $E$  being a tail. Further characterizations of the case when  $E$  is a tail are given. (Received February 28, 2009)