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**Ian Goulden\*** (ipgoulden@uwaterloo.ca), 200 University Ave. W., Waterloo, Ontario N2L3G1, Canada, and **William Slofstra**. *Annular embeddings of permutations for arbitrary genus*.

In the symmetric group on a set of size  $2n$ , let  $\mathcal{P}_{2n}$  denote the conjugacy class of involutions with no fixed points (equivalently, we refer to these as “pairings”, since each disjoint cycle has length 2). Harer and Zagier explicitly determined the distribution of the number of disjoint cycles in the product of a fixed cycle of length  $2n$  and the elements of  $\mathcal{P}_{2n}$ . Their famous result has been reproved many times, primarily because it can be interpreted as the genus distribution for 2-cell embeddings in an orientable surface, of a graph with a single vertex attached to  $n$  loops. In this paper we give a new formula for the cycle distribution when a fixed permutation with two cycles (say the lengths are  $p, q$ , where  $p + q = 2n$ ) is multiplied by the elements of  $\mathcal{P}_{2n}$ . It can be interpreted as the genus distribution for 2-cell embeddings in an orientable surface, of a graph with two vertices, of degrees  $p$  and  $q$ . In terms of these graphs, the formula involves a parameter that allows us to specify, separately, the number of edges between the two vertices and the number of loops at each of the vertices. (Received August 28, 2009)