1052-20-141 **Dave Witte Morris*** (Dave.Morris@uleth.ca), Department of Math and Computer Science, University of Lethbridge, Lethbridge, AB T1K 3M4, Canada. *Survey of invariant orders on arithmetic groups.*

At present, there are more questions than answers about the existence of invariant orders on an arithmetic subgroup Γ of a simple $\mathbb{Q}$-group.

Definition. An order relation $\prec$ on Γ is *left-invariant* if

$$x \prec y \implies ax \prec ay \qquad \text{for all } a, x, y \in \Gamma.$$

If, in addition, $x \prec y \implies xa \prec ya$, then we say that $\prec$ is *bi-invariant.*

It is easy to construct nontrivial partial orders on Γ that are left-invariant: choose any nonempty subset S of Γ that is closed under multiplication, but does not contain 1, and define

$$x \prec y \quad \iff \quad x^{-1}y \in S.$$

Free groups (and many other hyperbolic groups) provide examples of arithmetic groups with bi-invariant order relations that are *total,* rather than merely partial. This means

$$\forall x, y \in \Gamma, \text{ either } \; x \prec y \; \text{ or } \; x \succ y \; \text{ or } \; x = y.$$

In contrast, we would like to prove in most cases that there do *not* exist either

- a partial order that is bi-invariant, rather than merely left-invariant, or
- a left-invariant order that is total, rather than merely partial.

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