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Xiaolong Han, Department of Mathematics, Wayne State University, Detroit, MI 48202, and
Guozhen Lu* (gzlu@math.wayne.edu), Department of Mathematics, Wayne State University,
Detroit, MI 48202. *A geometric covering lemma and nodal sets of eigenfunctions.*

In this talk we describe some new geometric covering lemma which is akin to but quite different from the classical Besicovitch covering lemma in the Euclidean spaces. More precisely, we prove the following type of covering lemma:

Let $n \geq 2$ and $\delta > 0$ be small enough, then given any finite collection of balls $\{B_\alpha\}_{\alpha \in I}$ in \mathbb{R}^n , one can select a subcollection B_1, \dots, B_N such that

$$\bigcup_{\alpha} B_\alpha \subset \bigcup_{i=1}^N (1 + \delta)B_i \quad (1)$$

and

$$\sum_{i=1}^N \chi_{B_i}(x) \leq c\delta^{-\frac{n}{2}} \log\left(\frac{1}{\delta}\right) \quad (2)$$

where c is a constant independent of δ . Such a covering lemma was first introduced by Chanillo and Muckenhoupt.

On the other hand, we apply this covering lemma to improve BMO and volume estimates of nodal sets for eigenfunctions u satisfying $\Delta u + \lambda u = 0$ on n -dimensional Riemannian manifolds when λ is large, based on the works of Donnelly and Fefferman, Chanillo and Muckenhoupt. We also improve the BMO estimates for the function $q = |\nabla u|^2 + \frac{\lambda}{n}u^2$. Our covering lemma sharpens substantially earlier results and is fairly close to the optimal one we can expect. (Received August 26, 2009)