1053-03-121 **Russell G. Miller*** (Russell.Miller@qc.cuny.edu), Mathematics Dept., 65-30 Kissena Blvd., Flushing, NY 11367. *Real computability and roots of polynomials.* Preliminary report.

In ordinary Turing computability on fields such as $\mathbb{Q}$, the question of finding a root of a given polynomial boils down to the question of whether the field contains such a root. If it does, then a simple search procedure through the field suffices to produce the root, yielding a constructive proof of its existence. If it does not, then of course there is no proof, constructive or otherwise, that it has a root. So the problem of finding a constructive proof is Turing-equivalent to the problem of finding a classical proof.

Blum, Shub, and Smale generalized the notion of Turing computability to arbitrary rings. Using their definition, we point out that the situation in the real numbers $\mathbb{R}$ is different. There is a straightforward real-computable procedure for deciding whether a polynomial in $\mathbb{R}[X]$ has a root in $\mathbb{R}$, but no real-computable function can produce a root for every polynomial which has one. Indeed, this remains true even when the machines are given the ability to find n-th roots of arbitrary positive real numbers. This distinguish the real numbers from Turing-computable (countable) fields, and offers possibilities for connections to constructive mathematics. (Received August 27, 2009)