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Bas Spitters* (spitters@cs.ru.nl), Department of Computer Science, P.O. Box 9010, NL-6500 GL, Nijmegen, Netherlands, and **Chris Heunen** and **Nicolaas Landsman**. *A topos for algebraic quantum theory*.

We relate algebraic quantum mechanics to topos theory, so as to construct new foundations for quantum logic and quantum spaces. Motivated by Bohr's idea that the empirical content of quantum physics is accessible only through classical physics, we show how a C*-algebra of observables A induces a topos $T(A)$ in which the amalgamation of all of its commutative subalgebras comprises a single commutative C*-algebra. According to constructive Gelfand duality, the latter has an internal spectrum $S(A)$ in $T(A)$, which in our approach plays the role of a quantum phase space. Thus we associate a "pointfree space" to a C*-algebra (a noncommutative space). In this setting, states on A become probability measures on $S(A)$, and self-adjoint elements of A define continuous functions from $S(A)$ to Scott's interval domain. Noting that open subsets of $S(A)$ correspond to propositions about the system, the pairing map that assigns a truth value to a state and a proposition assumes an extremely simple categorical form. Formulated in this way, the quantum theory defined by A is essentially turned into a classical theory, internal to the topos $T(A)$. <http://arxiv.org/abs/0709.4364> (Received September 08, 2009)