There is a classical determinant formula for the number of lattice paths in the plane that take only unit steps east and north and that avoid a given boundary. Since the 19th century it has been known that there is a much simpler formula in certain special cases, notably when the boundary is the line $x = ky$ where $k$ is a positive integer. We show that the natural generalization of this simple formula continues to hold when the line $x = ky$ is replaced by certain periodic staircase boundaries—but only under special conditions. We have proved our results both combinatorially and algebraically, and later, Irving and Rattan found yet another way to prove some of our theorems. It remains an open question what other classes of lattice paths admit similar simple formulas. (Received August 28, 2009)