An integral domain $D$ is weakly integrally closed if whenever $x$ is in the quotient field of $D$, and $J$ is a nonzero finitely generated ideal of $D$ such that $xJ$ is contained in $JJ$, then $x$ is in $D$. We define weakly integrally closed numerical monoids similarly. If a monoid algebra is weakly integrally closed, then so is the monoid. The characteristic function of a numerical monoid $M$ can be thought of as an infinite binary string $s(M)$. A pattern of finitely many 0s and 1s is called forbidden if whenever $s(M)$ contains it, then $M$ is not weakly integrally closed. The pattern 11011 is forbidden. We show that a numerical monoid $M$ is weakly integrally closed if and only if $s(M)$ contains no forbidden patterns. We also show that for every finite set $S$ of forbidden patterns, there exists a numerical monoid $M$ that is not weakly integrally closed and for which $s(M)$ contains no stretch (in a natural sense) of a pattern in $S$. In doing so, we answer the question proposed by Brewer and Richman as to whether or not weakly integrally closed numerical monoid algebras are characterized by the property that the binary string of the monoid does not contain the pattern 11011. (Received September 08, 2009)