We introduce a new method to construct a rough path above a $d$-dimensional fractional Brownian motion $B^H$ with any Hurst parameter $H \in (0,1)$. This method has been inspired by the approach of J. Unterberger, and it is based on the representation of the fractional Brownian motion as a Volterra Gaussian process

$$B^H_t = \int_0^t K_H(t, s) dW_s,$$

where $\{W_t\}$ is a $d$-dimensional standard Wiener process. The main idea is to define iterated integrals

$$\int_{s<u_1<\cdots<u_n<t} dB^{H,i_1}_{u_1} \cdots dB^{H,i_n}_{u_n}$$

for $0 \leq s < t \leq T$, $n \leq [1/H]$ and $i_1, \ldots, i_n \in \{1, \ldots, d\}$, in such a way that they satisfy the required properties of Hölder continuity, multiplicativity and geometricity. A compact and simple formula for these iterated integrals is given. The method can be extended to a general class of Gaussian Volterra processes. We will discuss how this construction allows us to solve stochastic differential equations driven by a fractional Brownian motion with any Hurst parameter. This is a joint work with Samy Tindel. (Received May 26, 2009)