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Let  $G$  be a group and let  $A$  be an infinite set of generators for  $G$ . The length of an element  $x \in G$  with respect to the generating set  $A$ , denoted  $\ell_A(x)$ , is the length of the shortest representation of  $x$  as a finite product of elements in  $A \cup A^{-1}$ . For every nonnegative integer  $r$ , the sphere  $S_A(r)$  is the set of all elements  $x \in G$  of length exactly  $r$ . It is proved that either  $|S_A(r)| = \infty$  for all  $r$ , or there exists a unique integer  $r$  such that  $S_A(r')$  is empty for all  $r' > r$ ,  $S_A(r')$  is infinite for all  $r' < r$ , and  $S_A(r)$  is nonempty. The integer  $r$  is called the *phase transition* of the pair  $(G, A)$  and the set  $S_A(r)$  is called the *transition set*. A complete description of phase transitions and transition sets can be given for the integers and for certain other abelian groups. (Received September 14, 2009)