Shirley Law and Nathan Reading* (nathan_reading@ncsu.edu). The Hopf algebra and lattice (and polytope?) of rectangulations.

Twisted Baxter permutations arose (via lattice theory), as a natural basis for a sub Hopf algebra of the Malvenuto-Reutenauer Hopf algebra of permutations. The starting point of this research is the project of finding an intrinsic description of the Hopf algebra of twisted Baxter permutations in terms of a set of combinatorial objects in bijection with twisted Baxter permutations. This is accomplished using the (diagonal) rectangulations studied by Ackerman, Barequet, and Pinter, which are closely related to the twin binary trees of Dulucq and Guibert. In addition to the Hopf-theoretic results, we obtain a combinatorial characterization of the natural lattice structure on rectangulations, analogous to the Tamari lattice of triangulations. It appears that the Hasse diagram of this lattice is the 1-skeleton of a polytope. (We are checking the details of a nice fiber-polytopes proof.) We also find a new explicit bijection between twisted Baxter permutations and the better-known Baxter permutations.

The main tools include a surjective map from permutations to rectangulations, as well as combinatorial and lattice-theoretic results on a related family of maps from permutations to triangulations. (Received January 26, 2010)