

1057-11-2

**Bruce Reznick\***, Department of Mathematics, University of Illinois, Urbana, IL. *The secret life of polynomial identities.*

Polynomial identities can reflect deeper mathematical phenomena. In this talk, I will discuss some of the stories behind the following three identities (and their relatives):

$$1024x^{10} + 1024y^{10} + (x + \sqrt{3}y)^{10} + (x - \sqrt{3}y)^{10} + (\sqrt{3}x + y)^{10} + (\sqrt{3}x - y)^{10} = 1512(x^2 + y^2)^5 \quad (1)$$

$$x^3 + y^3 = \left( \frac{x(x^3 + 2y^3)}{x^3 - y^3} \right)^3 + \left( \frac{y(y^3 + 2x^3)}{y^3 - x^3} \right)^3, \quad (2)$$

$$(x^2 + \sqrt{2}xy - y^2)^5 + (ix^2 - \sqrt{2}xy + iy^2)^5 + (-x^2 + \sqrt{2}xy + y^2)^5 + (-ix^2 - \sqrt{2}xy - iy^2)^5 = 0 \quad (3)$$

Equation (1) has roots in 19th century mathematics; (2) is due to Viète (1592); (3) was independently found by Desboves (1880) and Elkies (1995). Their stories involve algebra, analysis, number theory, combinatorics, geometry and numerical analysis. Fourteenth powers of polynomials will show up. (Received April 09, 2009)