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On the structure of pure O -sequences: unimodality, non-unimodality, and an interval conjecture.

This is a report on an upcoming monograph joint with M. Boij, J. Migliore, R. Miró-Roig and U. Nagel.

An *order ideal* is a collection, X , of monomials in a finite number of variables such that, whenever $M \in X$ and N divides M , $N \in X$. If all maximal (by divisibility) monomials of X have the same degree, X is *pure*. A *pure O -sequence* is the vector counting the monomials of X in each degree.

The study of pure O -sequences began with one of the early works of Stanley in this area, and has since played a significant role in at least three theories: those of simplicial complexes, level algebras, and matroids.

Using both algebraic and combinatorial techniques, our monograph studies the structure of a pure O -sequence. In particular, it includes the following topics: a characterization of the first half of a pure O -sequence; a study of (the failing of) the unimodality property; the enumeration problem; a study of the Interval Conjecture for Pure O -sequences, and a connection with Stanley's conjecture on matroid h -vectors; an analysis of the role played by the Weak Lefschetz Property.

This talk will consist of a brief general overview of the theory of pure O -sequences, with some focus on unimodality and the ICP. (Received January 15, 2010)