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Bowling Green State University, Bowling Green, OH 43403. *A Tetrachotomy for Certain Algebras  
Containing the Disc Algebra.*

We will answer a question raised by Joseph Cima. Let  $D$  denote the open unit disc in the plane, and let  $A(D)$  denote the disc algebra. A theorem of E. M. Čirka asserts that if  $f$  is a function in  $C(\overline{D})$  and  $f$  is harmonic but nonholomorphic on  $D$ , then the uniformly closed subalgebra  $A(D)[f]$  of  $C(\overline{D})$  generated by  $A(D)$  and  $f$  is equal to  $C(\overline{D})$ . An analogous result for  $H^\infty(D)$  was proved by Sheldon Axler and Allen Shields: If  $f$  is a bounded function on  $D$  that is harmonic but nonholomorphic, then the uniformly closed subalgebra  $H^\infty(D)[f]$  of  $L^\infty(D)$  generated by  $H^\infty(D)$  and  $f$  contains  $C(\overline{D})$ .

Taken together these two theorems suggest that perhaps the inclusion  $A(D)[f] \supset C(\overline{D})$  holds whenever  $f$  is a bounded harmonic nonholomorphic function on  $D$ . However, this is false; it is not even true that  $A(D)[f, \bar{f}] \supset C(\overline{D})$  whenever  $f \in H^\infty(D)$ . This led Cima to ask which continuous functions are in  $A(D)[f]$  or  $A(D)[f, \bar{f}]$  when the inclusion  $A(D)[f] \supset C(\overline{D})$  or  $A(D)[f, \bar{f}] \supset C(\overline{D})$  fails. We will answer this question for  $A(D)[f, \bar{f}]$ . (Received December 03, 2009)