We will answer a question raised by Joseph Cima. Let $D$ denote the open unit disc in the plane, and let $A(D)$ denote the disc algebra. A theorem of E. M. Čirka asserts that if $f$ is a function in $C(D)$ and $f$ is harmonic but nonholomorphic on $D$, then the uniformly closed subalgebra $A(D)[f]$ of $C(D)$ generated by $A(D)$ and $f$ is equal to $C(D)$. An analogous result for $H^\infty(D)$ was proved by Sheldon Axler and Allen Shields: If $f$ is a bounded function on $D$ that is harmonic but nonholomorphic, then the uniformly closed subalgebra $H^\infty(D)[f]$ of $L^\infty(D)$ generated by $H^\infty(D)$ and $f$ contains $C(D)$.

Taken together these two theorems suggest that perhaps the inclusion $A(D)[f] \supset C(D)$ holds whenever $f$ is a bounded harmonic nonholomorphic function on $D$. However, this is false; it is not even true that $A(D)[f, \bar{f}] \supset C(D)$ whenever $f \in H^\infty(D)$. This led Cima to ask which continuous functions are in $A(D)[f]$ or $A(D)[f, \bar{f}]$ when the inclusion $A(D)[f] \supset C(D)$ or $A(D)[f, \bar{f}] \supset C(D)$ fails. We will answer this question for $A(D)[f, \bar{f}]$. (Received December 03, 2009)