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Let f be a polynomial of degree $n \geq 2$ with $f(0) = 0$ and $f'(0) = 1$. Smale conjectured that there is a critical point ζ of f , that is, a zero of f' , such that $|f(\zeta)/\zeta| \leq 1 - 1/n$. Addressing a special case of this problem, we prove that there is a critical point ζ of f with $|f(\zeta)/\zeta| \leq 1/2$ provided that the critical points of f lie in the sector $\{re^{i\theta} : r > 0, |\theta| \leq \pi/6\}$, and $|f(\zeta)/\zeta| \leq 2/3$ if they lie in the union of two rays from the origin to infinity (for example, the real axis), or in the union of the two rays $\{1 + re^{\pm i\theta} : r \geq 0\}$, where $0 < \theta \leq \pi/2$. We identify the cases of equality. The best previously known degree-independent upper bound for the case of real critical points was $e - 2$, due to Sheil-Small and to Rahman and Schmeisser. (Received January 25, 2010)