1057-42-213 **Leonid Slavin*** (`leonid.slavin@uc.edu`) and **Vasily Vasyunin**. *The embedding $BMO \subset L^p_{loc}$ and sharp equivalence of BMO norms.*

The space $\mathrm{BMO}_p(\mathbb{R})$ is defined, for all $p \geq 1$, by

$$\mathrm{BMO}_p = \left\{ \varphi \in L^1_{loc} : \sup_{\text{interval } Q} \langle |\varphi - \langle\varphi\rangle_Q|^p| \rangle_Q \leq C^p < \infty \right\},$$

with $\langle\varphi\rangle_Q \stackrel{\text{def}}{=} \frac{1}{|Q|}\int_Q \varphi$ and the best such C being the corresponding norm. It is known that the norms are equivalent for all p, with one direction following from Hölder's inequality and the other usually regarded as a consequence of the John–Nirenberg inequality. However, the constants of this equivalence are not known.

We find the explicit upper and lower Bellman functions for the embedding $\mathrm{BMO}_2 \subset L^p_{loc}$ thus establishing the sharp embedding constant. As a consequence, we can relate, sharply, all BMO_p norms to the BMO_2 norm. The proof depends on solving a Monge–Ampeère equation on a non-convex domain, coupled with a delicate induction argument. As an integral part of the solution, we construct the Bellman foliation of the domain, yielding the extremizers in the inequalities being proved. This is joint work with V. Vasyunin. (Received January 22, 2010)