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J. Sivaloganathan (masjs@bath.ac.uk), Department of Mathematical Sciences, University of Bath, and **S. J. Spector*** (sspector@math.siu.edu), Department of Mathematics, Southern Illinois University. *On the Symmetry of Energy-Minimizing Deformations in Nonlinear Elasticity.*

Consider a homogeneous, isotropic, hyperelastic body that occupies an n -dimensional annulus in its reference state and is subject to radially symmetric displacement boundary conditions on its inner or outer boundary. We show that for a large class of polyconvex stored-energy functions the radial minimizer of this problem is an absolute minimizer of the energy. The key ingredient is a new radial-symmetrization procedure that yields a one-to-one map.

For the pure displacement boundary-value problem, the radial symmetrization of an orientation preserving $\mathbf{u} : A \rightarrow A^*$ between annuli A and A^* is the deformation

$$\mathbf{u}_r(\mathbf{x}) = \frac{r(R)}{R} \mathbf{x}, \quad R = |\mathbf{x}|,$$

that maps each sphere $S_R \subset A$, of radius $R > 0$, centered at the origin into another such sphere $S_r = \mathbf{u}_r(S_R) \subset A^*$ that encloses the same volume as $\mathbf{u}(S_R)$. Since the volumes enclosed by the two surfaces are equal, the isoperimetric inequality yields

$$\text{area}(\mathbf{u}_r(S_R)) \leq \text{area}(\mathbf{u}(S_R)).$$

The equality of the volumes together with this reduction in surface area gives rise to a reduction in total energy for many of the constitutive relations used in elasticity. (Received December 28, 2009)