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Tao Jiang, Zevi Miller and Dan Pritikin* (pridikd@muohio.edu), Department of Mathematics, Miami University, Oxford, OH 45056. *The Steiner problem in the hypercube.*

Let S be a set of vertices in the hypercube Q_n . We study the *Steiner cost* of S , denoted $cost(S)$, the minimum number of edges among connected subgraphs of Q_n containing S . We obtain the the following results by probabilistic methods.

(1) If $|S| = k$, then $cost(S) \leq \frac{1}{3}(k + 1 + \ln(k - 1))n$.

(2) The above bound is nearly best possible for a certain range in k as follows. Let $\epsilon > 0$ be a fixed small real number, and let n be sufficiently large as a function of ϵ . Further let k lie in the range $K_1 \leq k \leq K_2 c^n$, where K_1 , K_2 , and c constants which depend only on ϵ , with $1 < c < 2$. Then there exist sets S in Q_n of size k such that $cost(S) \geq (\frac{1}{3} - \epsilon)kn$.

(3) In some random sense, these Steiner costs are tightly concentrated about their mean.

The work naturally generalizes to a "squashed cube" setting concerning optimal interconnection of a set of subcubes of Q_n instead of handling only the case in which each subcube has dimension 0 (i.e., is a single vertex). (Received February 12, 2010)