

1058-05-196

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Let $\nu(G)$ denote the maximum number of edge-disjoint triangles in a graph G and $\tau(G)$ denote the minimum number of edges covering all triangles of G . Motivated by Tuza's conjecture that $\tau(G) \leq 2\nu(G)$ for every graph G , we sharpen two known results in this direction. Tuza proved that his conjecture holds for planar graphs. This result is sharp, since for the graph K_4 we have $\nu(K_4) = 1$ and $\tau(K_4) = \tau^*(K_4) = 2$. We prove that for every planar graph G with no K_4 , $\tau(G) \leq 1.5\nu(G)$. This bound is attained at the 5-wheel. It implies that every planar graph G with $\tau(G)$ "close" to $2\nu(G)$ contains "many" K_4 -subgraphs.

Let $\tau^*(G)$ denote the minimum total weight of a fractional covering of its triangles by edges. Krivelevich proved that $\tau^*(G) \leq 2\nu(G)$ for every graph G . We refine this result by showing that if a graph G has $\tau^*(G) \geq 2\nu(G) - x$, then G contains $\nu(G) - \lfloor 10x \rfloor$ edge-disjoint K_4 -subgraphs plus $\lfloor 10x \rfloor$ additional edge-disjoint triangles. Our proof also yields that $\tau^*(G) \leq 1.8\nu(G)$ for each K_4 -free graph G . (Received February 16, 2010)