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Walter Taylor* (walter.taylor@colorado.edu), Professor Walter Taylor, University of Colorado, Boulder, CO 80309-0395. *Approximate satisfaction of equations.*

For a topological algebra \mathbf{A} based on a metric space $A = (A, d)$, and for Σ a set of equations, we define $\lambda_{\mathbf{A}}(\Sigma)$ to be the sup of $d(\sigma^{\mathbf{A}}(a), \tau^{\mathbf{A}}(a))$, over all $\sigma \approx \tau \in \Sigma$ and over all assignments a of A -values to variables. Then, for a metric space A , $\lambda_A(\Sigma)$ is defined as the inf of $\lambda_{\mathbf{A}}(\Sigma)$, over all topological algebras on A . $\lambda_A(\Sigma)$ measures how far the equations Σ must deviate from being satisfied on A with continuous operations.

λ_A is not a topological invariant of A , but depends heavily on the metric.

For certain A , such as a closed unit interval, we have: if $\lambda_A(\Gamma \times \Delta) < \varepsilon$, then $\lambda_A(\Gamma) < 4\varepsilon$ or $\lambda_A(\Delta) < 4\varepsilon$. This is an analog to the known result for topological algebras that if $[0, 1]$ models $\Gamma \times \Delta$, then $[0, 1]$ models Γ or $[0, 1]$ models Δ .

Let A be the realization of a finite simplicial complex, and let α be a computable real number. By simplicial approximation, the set of all finite Σ (in a fixed alphabet) such that $\lambda_A(\Sigma) < \alpha$ is recursively enumerable. (Received December 01, 2009)