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**Scott T Chapman\*** ([scott.chapman@shsu.edu](mailto:scott.chapman@shsu.edu)), Sam Houston State University, Department of Mathematics and Statistics, Box 2206, Huntsville, TX 77341-2206, and **William W Smith** ([wsmith@email.unc.edu](mailto:wsmith@email.unc.edu)), University of North Carolina at Chapel Hill, Department of Mathematics, Phillips Hall, Chapel Hill, NC 27599-3250. *Erdős-Zaks All Divisor Sets.*

Let  $\mathbb{Z}_n$  be the finite cyclic group of order  $n$  and  $S \subseteq \mathbb{Z}_n$ . We examine the factorization properties of the Block Monoid  $\mathcal{B}(\mathbb{Z}_n, S)$  when  $S$  is constructed using a method inspired by a 1990 paper of Erdős and Zaks. For such a set  $S$ , we develop an algorithm in Section 2 to produce and order a set  $\{\mathfrak{M}_i\}_{i=1}^{n-1}$  which contains all the non-primary irreducible Blocks (or atoms) of  $\mathcal{B}(\mathbb{Z}_n, S)$  and use this set to argue that  $\mathcal{B}(\mathbb{Z}_n, S)$  is weakly half-factorial. After developing some basic properties of the set  $\{\mathfrak{M}_i\}_{i=1}^{n-1}$ , we examine in Section 3 the connection between these irreducible blocks and the Erdős-Zaks notion of “splittable sets.” In particular, the Erdős-Zaks notion of “irreducible” does not match the classic notion of “irreducible” for the commutative cancellative monoids  $\mathcal{B}(\mathbb{Z}_n, S)$ . We close with a detailed discussion of the special properties of the blocks  $\mathfrak{M}_1$  with an emphasis on the case where the exponents of  $\mathfrak{M}_1$  take on extreme values. (Received February 16, 2010)