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**Patricia Cahn\*** ([patricia.cahn@dartmouth.edu](mailto:patricia.cahn@dartmouth.edu)), 6188 Kemeny Hall, Dartmouth College, Hanover, NH 03755. *A Generalization of the Turaev Cobracket and the Minimal Self-Intersection Number*. Preliminary report.

Goldman and Turaev constructed a Lie bialgebra structure on the free  $\mathbb{Z}$ -module generated by free homotopy classes of loops on a surface. The Turaev cobracket  $\Delta(\alpha)$  gives a lower bound on the minimal number of self-intersection points of a loop in a given homotopy class. Chas found examples which prove that this lower bound is not sharp. In particular, she constructed a class  $\alpha$  with  $\Delta(\alpha) = 0$ , but which is not realized by a power of a simple loop. This disproves Turaev's conjecture that  $\Delta(\alpha) = 0$  if and only if  $\alpha$  can be realized by a power of a simple loop. We introduce an operation  $\mu$ , defined in the spirit of the Andersen-Mattes-Reshetikhin algebra of chord diagrams. The Turaev cobracket factors through  $\mu$ , and  $\mu$  also gives a lower bound on the minimal number of self-intersection points of a loop in a given homotopy class. We show that this lower bound is sharp for homotopy classes  $\alpha$  such that  $\alpha \neq \beta^i$  for  $|i| > 1$ . We also show that  $\mu(\alpha) = 0$  if and only if  $\alpha$  can be realized by a power of a simple loop, thus showing that a statement similar to Turaev's conjecture is true. (Received February 02, 2010)