We present that the operator norm on weighted Lebesgue space $L^2(w)$ of the commutators of the Hilbert, Riesz and Beurling transforms with a $BMO$ function $b$ depends quadratically on the $A_2$-characteristic of the weight, as opposed to the linear dependence known to hold for the operators themselves. It is known that the operator norms of these commutators can be controlled by the norm of the commutator with appropriate Haar shift operators, and we prove the estimate for these commutators. For the shift operator corresponding to the Hilbert transform we use Bellman function methods, however there is now a general theorem for a class of Haar shift operators that can be used instead to deduce similar results. We invoke this general theorem to obtain the corresponding result for the Riesz transforms and the Beurling-Ahlfors operator. We can then extrapolate to $L^p(w)$, and the results are sharp for $1 < p < \infty$. We also present the examples that show the quadratic bounds are sharp for the commutators of the Hilbert, Riesz and Beurling transforms. (Received January 10, 2010)