1059-42-186 **Leonid Slavin*** (leonid.slavin@uc.edu) and **Vasily Vasyunin**. *The embedding* BMO $\subset L^p_{loc}$ *and sharp equivalence of BMO norms.*

The space $\mathrm{BMO}_p(\mathbb{R})$ is defined, for all $p \geq 1$, by

$$\mathrm{BMO}_p = \left\{ \varphi \in L^1_{loc} : \sup_{\text{interval } Q} \langle |\varphi - \langle \varphi \rangle_Q|^p | \rangle_Q \leq C^p < \infty \right\},$$

with $\langle \varphi \rangle_Q \stackrel{\text{def}}{=} \frac{1}{|Q|} \int_Q \varphi$ and the best such C being the corresponding norm. It is known that the norms are equivalent for all p, with one direction following from Hölder's inequality and the other usually regarded as a consequence of the John–Nirenberg inequality. However, the constants of this equivalence are not known.

We find the explicit upper and lower Bellman functions for the embedding $\mathrm{BMO}_2 \subset L^p_{loc}$ thus establishing the sharp embedding constant. As a consequence, we can relate, sharply, all BMO_p norms to the BMO_2 norm. The proof depends on solving a Monge–Ampère equation on a non-convex domain, coupled with a delicate induction argument. As an integral part of the solution, we construct the Bellman foliation of the domain, yielding the extremizers in the inequalities being proved. (Received February 23, 2010)