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**Le Nhat Tuan\*** (kid10462000@yahoo.com), 609 South Bruce St, Anaheim, CA 92804. *On the extremal problem of Polya.* Preliminary report.

The notion of transfinite diameter of planar sets was introduced by M. Fekete around 1920s. This concept plays an important role in the classical complex analysis and is related to other well-known concepts such as the logarithmic capacity and Chebyshev polynomials. For each  $n \geq 3$ , the  $n$ -diameter  $d_n(E)$  of  $E$  is given by  $d_n(E) = \max \left\{ \prod_{1 \leq i < j \leq n} |z_i - z_j|^{2/n(n-1)} \right\}$

The following is the extremal problem of G. Polya: among all  $n$ -tuples  $E = \{z_1, z_2, \dots, z_n\}$  with  $|z_i| \leq 1$ , find one with the largest  $n$ -diameter. The solution of this problem, attributed to Polya, is  $d_n(E) \leq n^{\frac{1}{n-1}}$  and the equality holds for  $n$ -tuples of equally spaced points on the boundary of  $D$ . While investigating the transfinite diameter of sets of constant width, Prof. Zair Ibragimov was led to the following weaker version of Polya's problem: among all  $n$ -tuples  $E = \{z_1, z_2, \dots, z_n\}$  with  $|z_i - z_j| \leq 2$ , find one with the largest  $n$ -diameter. He conjectured that the maximum is reached when this  $n$ -gon is regular, at least when  $n$  is odd. In this paper, I will present my solution of Ibragimov's problem for the case  $n = 5$  and an "almost completed" approach for  $n = 7$ . (Received February 17, 2010)