We consider two operations on the edge of an embedded (i.e. ribbon) graph: giving a half-twist to the edge and taking the partial dual with respect to the edge. These two operations give rise to an action of $S_3^{e(G)}$, the ribbon group of $G$, on $G$. We show that this ribbon group action gives a complete characterization of duality in that if $G$ is any cellularly embedded graph with medial graph $G_m$, then the orbit of $G$ under the group action is precisely the set of all graphs with medial graphs isomorphic (as abstract graphs) to $G_m$. We then show how this group action leads to a deeper understanding of the properties of, and relationships between, various graph polynomials such as the generalized transition polynomial, an extension of the Penrose polynomial to embedded graphs, and the topological Tutte polynomials of Las Vergnas and also Bollobás and Riordan, as well as various knot and link invariants.

This work is motivated by a problem in self-assembling DNA nanostructures, and the results include a possible self-assembly design strategy based on a medial graph template. (Received March 02, 2010)