1060-16-5 **Aleks Kleyn*** (Aleks_Kleyn@MailAPS.org), 2709 Brown str, Brooklyn, NY 11235. *The Gâteaux Derivative and Integral over Division Ring.*

Let $Z(D)$ be center of division ring D. Map

$$f : D \to D$$

is linear if for any $a, b \in D$ and any $c \in Z(D)$

$$f(a + b) = f(a) + f(b)$$

$$f(ca) = cf(a)$$

Map

$$f : D \to D$$

is called differentiable in the Gâteaux sense,if

$$f(x + a) - f(x) = \partial f(x)(a) + o(a)$$

where the Gâteaux derivative $\partial f(x)$ of map f is linear map of increment a and o is such continuous map that

$$\lim_{a \to 0} \frac{|o(a)|}{|a|} = 0$$

For instance

$$\partial(x^2)(h) = xh + hx$$

$$\partial(x^{-1})(h) = -x^{-1}hx^{-1}$$

Assuming that we defined the Gâteaux derivative $\partial^{n-1} f(x)$ of order $n-1$,we define

$$\partial^n f(x)(a_1; ...; a_n) = \partial(\partial^{n-1} f(x)(a_1; ...; a_{n-1}))(a_n)$$

the Gâteaux derivative of order n of map f. When $h_1 = ... = h_n = h$, we assume

$$\partial^n f(x)(h) = \partial^n f(x)(h_1; ...; h_n)$$

Function $f(x)$ has Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} (n!)^{-1} \partial^n f(x_0)(x - x_0)$$

Differential equation over division ring

$$\partial(y)(h) = hx^2 + xhx + x^2 h$$
$$y(0) = 0$$

has solution

$$y = x^3$$

The solution of differential equation

$$\partial(y)(h) = \frac{1}{2}(yh + hy)$$
$$y(0) = 1$$

is exponent $y = e^x$ that has following Taylor series expansion

$$e^x = \sum_{n=0}^{\infty} (n!)^{-1} x^n$$

The equation

$$e^{a+b} = e^a e^b$$

is true iff $ab = ba$ (Received August 25, 2009)