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Murray R. Bremner* (bremner@math.usask.ca), Mathematics and Statistics, University of Saskatchewan, Saskatoon, SK S7N 5E6, Canada, and **Hader A. Elgendy**. *Universal enveloping algebras of n -Lie algebras*. Preliminary report.

An n -Lie algebra is a vector space L with a multilinear product $L^n \rightarrow L$ satisfying n -ary anticommutativity and the n -ary Jacobi identity. These structures were introduced by Filippov in 1985; in the case $n = 2$ we obtain the definition of a Lie algebra. Ling proved in 1993 that for $n \geq 3$, there is only one simple finite-dimensional n -Lie algebra; it has dimension $n+1$ and generalizes the cross product on \mathbb{R}^3 to an n -ary product on \mathbb{R}^{n+1} . We study representations of n -Lie algebras in associative algebras by means of the n -ary alternating sum. This leads to the problem of determining the universal enveloping algebra $U(L)$ of an n -Lie algebra L . Using the theory of Gröbner bases, we determine a basis for $U(L)$ when n is even and L is the simple n -Lie algebra. As a corollary, we find that the natural map from L to $U(L)$ is injective; this is a partial generalization of the PBW theorem to n -Lie algebras. (For odd n the situation is much more complicated.) We obtain a new proof of some results of Pozhidaev from 2003 for $n \leq 6$, but our results seem to be new for $n \geq 8$. This is joint work with my Ph.D. student Hader Elgendy. (Received March 08, 2010)