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**Maciej Niebrzydowski** and **Jozef H. Przytycki\*** (przytyck@gwu.edu), Department of Mathematics, George Washington, Monroe Hall, Room 240 2115 G Street NW, Washington, DC 20052. *The second quandle homology of the Takasaki quandle of an odd abelian group is an exterior square of the group.*

M. Takasaki introduced the notion of kei (involutive quandle) in 1942. His main example was the quandle of an abelian group  $T(G)$  with  $a * b = 2b - a$ , which we call a Takasaki quandle. We prove that if  $G$  is an abelian group of odd order then the second quandle homology  $H_2^Q(T(G))$  is isomorphic to  $G \wedge G$  where  $\wedge$  is the exterior product. In particular, for  $G = Z_k^n$ ,  $k$  odd we have  $H_2^Q(T(Z_k^n)) = Z_k^{n(n-1/2)}$ . We start our proof by constructing Cayley graph and Cayley 2-complex of  $T(G)$  in such a way that the first homology of the complex is the second homology of  $T(G)$ . We choose a spanning tree for the Cayley graph and contract it. The homological result is the group  $Z(G \times G)$  divided by relations  $[x, x] = 0$  (this corresponds to the fact that in quandle homology we nullify degenerate elements),  $[0, x] = 0$  (elements of a chosen spanning tree are equal to zero) and  $[x, z] + [z, y] = [x, z - y + x] + [z - y + x, y]$ . Then we prove that for  $G$  generated by 2 elements the main result holds, in particular that  $[x, y] = -[y, x]$ . After some algebraic manipulations we obtain generally  $G \wedge G$ . Our result can be directly applied to classical links as 2-(co)cycles give link invariants. They also can be used to produce new nontrivial quandles. (Received March 31, 2010)