

1060-91-163

**Peter Friz, Stefan Gerhold, Archil Gulisashvili and Stephan Sturm\***

([ss Sturm@princeton.edu](mailto:ss Sturm@princeton.edu)), 116 Sherrerd Hall, Princeton University, Princeton, NJ 08544. *On refined volatility smile expansion in the Heston model.*

It is known that Heston's stochastic volatility model exhibits moment explosion, and that the critical moment  $s^*$  can be obtained by solving (numerically) a simple equation. This yields a leading order expansion for the implied volatility at large strikes:  $\sigma_{BS}(k, T)^2 T \sim \Psi(s^* - 1) \times k$  (Roger Lee's moment formula). Motivated by recent "tail-wing" refinements of this moment formula, we first derive a novel tail expansion for the Heston density, and then show the validity of a refined expansion of the type  $\sigma_{BS}(k, T)^2 T = (\beta_1 k^{1/2} + \beta_2 + \dots)^2$ , where all constants are explicitly known as functions of  $s^*$ , the Heston model parameters, spot vol and maturity  $T$ . In the case of the "zero-correlation" Heston model such an expansion was derived by Gulisashvili and Stein. Our methods and results may prove useful beyond the Heston model: the entire quantitative analysis is based on affine principles; at no point do we need knowledge of the closed form expression of the Fourier transform of  $\log S_T$ . Secondly, our analysis reveals a new parameter ("*critical slope*"), defined in a model free manner, which drives the second and higher order terms in tail- and implied volatility expansions. (Received March 29, 2010)