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Jasun Gong*, Department of Mathematics, 301 Thackeray Hall, University of Pittsburgh,
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This is joint work with P. Hajlasz, P. Koskela, and X. Zhong.

Let $p > n$. Suppose that a domain Ω in \mathbf{R}^n has the $W^{1,p}$ -extension property; that is, there is a bounded linear operator

$$E : W^{1,p}(\Omega) \rightarrow W^{1,p}(\mathbf{R}^n)$$

so that $Ef|_{\Omega} = f$. It is clear that such domains satisfy the Sobolev imbedding theorem

$$|u(x) - u(y)| \leq C \|\nabla u\|_p |x - y|^{1-n/p}.$$

Here we are interested in the converse direction, namely

Theorem. Let Ω be a domain in \mathbf{R}^n and let $p > n$. The following are equivalent:

1. Ω satisfies the Sobolev imbedding theorem;
2. Ω has the $W^{1,q}$ -extension property, for each $q \geq p$;
3. For each $m \in \mathbf{N}$, Ω has the $W^{m,q}$ -extension property, for each $q \geq p$; that is, there exists a bounded linear operator $E_m : W^{m,p}(\Omega) \rightarrow W^{m,p}(\mathbf{R}^n)$ so that $E_m f|_{\Omega} = f$.

If time permits, we will also discuss the borderline cases $p = \infty$ and $p = n$. In the first case, our main result recovers a classical theorem of H. Whitney. (Received August 10, 2010)