We investigate the relationship, in various contexts, between a closed geodesic with self-intersection number \( k \) (for brevity, called a \( k \)-geodesic) and its length. We show that for a fixed compact hyperbolic surface, the short \( k \)-geodesics grow like the square root of \( k \). On the other hand, if the fixed hyperbolic surface has a cusp and is not the punctured disc, then the short \( k \)-geodesics grow logarithmically.

The length of a \( k \)-geodesic on any hyperbolic surface is known to be bounded from below by a constant that goes to infinity with \( k \). In this paper, we show that the optimal constants \( \{M_k\} \) grow like \( \log k \). Moreover, we show that for each natural number \( k \), there exists a hyperbolic surface where the constant \( M_k \) is realized as the length of a \( k \)-geodesic. This was previously known for \( k = 1 \), where the figure eight on the thrice punctured sphere is the shortest non-simple closed geodesic. (Received June 03, 2010)