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**David Cruz-Uribe** and **Cristian Rios\*** ([crios@ucalgary.ca](mailto:crios@ucalgary.ca)), 2500 University Drive NW, University of Calgary, Department of Mathematics, Calgary, AB T2N 1N4, Canada. *The Kato Problem for  $A_2$ -Elliptic Operators.*

Given a weight  $w$  in the Muckenhoupt class  $A_2$ , let  $\mathbf{A}$  be an  $n \times n$  matrix of complex-valued measurable functions such that for some  $0 < \lambda < \Lambda < \infty$ , and all  $\xi, \nu \in \mathbb{R}^n$ ,

$$\begin{cases} \lambda w(x) |\xi|^2 \leq \operatorname{Re} \langle \mathbf{A}\xi, \xi \rangle, \\ |\langle \mathbf{A}\xi, \eta \rangle| \leq \Lambda w(x) |\xi| |\eta| \end{cases}$$

Since  $w$  and  $w^{-1}$  can be unbounded, in general  $\mathbf{A}$  is a degenerate elliptic matrix. We define the second order elliptic operator  $\mathcal{L}_w = -w^{-1} \operatorname{div} \mathbf{A} \nabla$ . We show that the Kato problem for uniformly elliptic operators extends to this degenerate case. More precisely, the domain of  $\mathcal{L}_w^{1/2}$  is  $H^1(w)$ , and for all  $f \in H^1(w)$ ,

$$\|\mathcal{L}_w^{1/2} f\|_{L^2(w)} \approx \|\nabla f\|_{L^2(w)}.$$

The proof follows the scheme of the uniformly elliptic case proof due to Auscher, Hofmann, McIntosh, and Tchamitcian, but with significant differences and obstacles due to the weighted setting. (Received August 08, 2010)