1062-46-81 **Lawrence A. Harris***, Mathematics Department, University of Kentucky, Lexington, KY 40506-0027, and **Clifford J. Earle**, Mathematics Department, Cornell University, Ithaca, NY 14853-4201. *Inequalities for the Carathéodory and Poincaré metrics in open unit balls.*

Let Δ be the open unit disc of the complex plane and let ρ denote the Poincaré metric on Δ. It is shown in a previous paper by the authors and others that

$$|a-b| \le 2\tanh\frac{\rho(a,b)}{2} \quad \text{for all } a,b\in\Delta,$$

with equality if and only if $a=\pm b$. We consider the more general case where Δ is replaced by the open unit ball B of a complex Banach space X. We show that if d is any metric on B satisfying $\rho(\ell(a),\ell(b)) \le d(a,b)$ for all $a,b\in B$ and all continuous linear functionals ℓ on X of norm 1, then

$$\|a-b\| \le 2\tanh\frac{d(a,b)}{2} \quad \text{for all } a,b\in B. \tag{1}$$

For example, d could be any metric of a Schwarz-Pick system such as the Carathéodory or Kobayashi metric.

We obtain two necessary and sufficient conditions for equality to hold in (1) and then focus on determining spaces X where this implies that $a=\pm b$. Every Hilbert space has this property. If the open unit ball of a space with this property is a homogeneous domain, then the space must be a Hilbert space. We obtain a distortion form of the above result for Hilbert spaces and deduce an analogous form for real hyperbolic spaces. (Received July 27, 2010)