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Paul Shafer* (pshafer@math.cornell.edu), Department of Mathematics, 310 Malott Hall, Cornell University, Ithaca, NY 14853. *Birkhoff's theorem and reverse mathematics*. Preliminary report.

Three key theorems of finite matching theory are Hall's theorem determining when a bipartite graph has a perfect matching, König's duality theorem equating the maximum cardinality of a matching in a bipartite graph to the minimum cardinality of a cover in that graph, and Birkhoff's theorem decomposing an $n \times n$ doubly stochastic matrix (i.e., a matrix in which every row and column sums to one) into a convex combination of permutation matrices. In the finite case, Hall's theorem and König's duality theorem each have an easy proof from the other, and both theorems easily imply Birkhoff's theorem. The relations among these theorems are more colorful when generalized to countable cases and put in the context of reverse mathematics. Hirst proved that one countable version of Hall's theorem is equivalent to the system WKL_0 and that another is equivalent to the stronger system ACA_0 . Aharoni, Magidor, Shore, and Simpson proved that a countable version of König's duality theorem is equivalent to the even stronger system ATR_0 . We continue this program by analyzing, in the context of reverse mathematics, a countable version of Birkhoff's theorem originally due to Isbell. (Received September 12, 2010)