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The block-cutpoint graph $BG(G)$ of a graph G is a tree with bipartite sets that are the cut-vertices of G and the blocks (maximal biconnected components) of G , respectively. An edge clique-cover of G is a set of cliques $X \subseteq G$ whose graph union is G . Motivated by the study of random intersection graphs, we define graph weight as $\text{wt}(G) = \min_{\mathcal{S}} \sum_{X \in \mathcal{S}} (|V(X)| - 1)$, where \mathcal{S} ranges over all edge clique-covers of G . We characterize the least-weight supergraphs of G in two ways: by partitions of $BG(G)$ into edge-disjoint subtrees whose leaves have no cut-vertices as external neighbors, and in terms of partitions of the neighborhoods of each cut-vertex in $BG(G)$. As a result the least-weight supergraphs can be encoded using Touchard polynomials. Determining $\text{wt}(G)$ for K_4 -free graphs is equivalent to the well-known extremal question of finding the maximum number of edge-disjoint triangles in G , and we prove that a K_4 -free graph G with $\lfloor n^2/4 \rfloor + m$ edges has at least m edge-disjoint triangles when m close is to $n^2/12$. (Received September 12, 2010)