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Brian Harbourne* (bharbour@math.unl.edu), Department of Mathematics, University of Nebraska-Lincoln, Lincoln, NE 68588-0130. *Fat points, graded Betti numbers and geometry.*

Let $p_1, \dots, p_r \in \mathbf{P}^2$ be generic points (and let $\pi : X \rightarrow \mathbf{P}^2$ be obtained by blowing up the points p_i). The Hilbert function and graded Betti numbers of the ideal of every fat point subscheme $Z = \sum_i m_i p_i$ are known if $r < 9$. When $r > 9$ neither are known in general, but there is a conjecture for the Hilbert function. If $r = 9$, then the Hilbert function is known, but not in general the graded Betti numbers. A geometric way to study the graded Betti numbers, due to Fitchett, involves the splitting $(\pi^* \Omega_{\mathbf{P}^2})|_E = \mathcal{O}_E(-a) \oplus \mathcal{O}_E(-b)$ for exceptional curves E on X . When $r = 9$, recent joint work with A. Gimigliano and M. Idà suggests the following conjecture: $|b - a| \geq 2$ if and only if $E + L$ is a semi-adjoint (i.e, there is a divisor A such that $E + L + K_X = 2A$, where L is the pullback to X of a line in \mathbf{P}^2), in which case $|b - a| = 2$. This conjecture, if true, determines the graded Betti numbers for all Z in all degrees but one when $r = 9$. (Received September 02, 2010)