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**Jianguo Cao** (jcao@nd.edu), Notre Dame, IN 46556, and **Jian Ge\*** (jge@nd.edu), 255 Hurley Hall, University of Notre Dame, Notre Dame, IN 46556. *A simple proof of Perelman's collapsing theorem for 3-manifolds.*

We will simplify earlier proofs of Perelman's collapsing theorem for 3-manifolds given by Shioya-Yamaguchi and Morgan-Tian. A version of Perelman's collapsing theorem states: "*Let  $\{M_i^3\}$  be a sequence of compact Riemannian 3-manifolds with curvature bounded from below by  $(-1)$  and  $\text{diam}(M_i^3) \geq c_0 > 0$ . Suppose that all unit metric balls in  $M_i^3$  have very small volume at most  $v_i \rightarrow 0$  as  $i \rightarrow \infty$  and suppose that either  $M_i^3$  is closed or has possibly convex incompressible toral boundary. Then  $M_i^3$  must be a graph-manifold for sufficiently large  $i$* ". This result can be viewed as an extension of the implicit function theorem. Among other things, we apply Perelman's critical point theory (e.g., multiple conic singularity theory and his fibration theory) to Alexandrov spaces to construct the desired local Seifert fibration structure on collapsed 3-manifolds.

The verification of Perelman's collapsing theorem is the last step of Perelman's proof of Thurston's Geometrization Conjecture. A version of Geometrization Conjecture asserts that any closed 3-manifold admits a *piecewise locally homogeneous metric*. Our proof of Perelman's collapsing theorem is accessible to advanced graduate students and non-experts. (Received September 09, 2010)