

1064-57-372

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Representation stability, configuration spaces, and points over finite fields.

Given a space X , the configuration space $\text{PConf}_n(X)$ parametrizes n -tuples of distinct points, while $\text{Conf}_n(X) := \text{PConf}_n(X)/S_n$ parametrizes n -element subsets of X . Arnold proved that the spaces $\text{Conf}_n(\mathbb{C})$ satisfy *homological stability*: for each i , $H_i(\text{Conf}_n(\mathbb{C})) \approx H_i(\text{Conf}_{n+1}(\mathbb{C}))$ for $n \gg i$.

Unfortunately stability fails for $\text{PConf}_n(\mathbb{C})$, so to talk about the “stable homology of $\text{PConf}_n(\mathbb{C})$ ”, B. Farb and I defined *representation stability*. We take the action of S_n into account and prove the description of $H_i(\text{PConf}_n(\mathbb{C}); \mathbb{Q})$ as an S_n -representation stabilizes. More recently I proved that $H_i(\text{PConf}_n(M); \mathbb{Q})$ is representation stable for *any* manifold. This implies that $H_i(\text{Conf}_n(M); \mathbb{Q})$ satisfies homological stability, previously known only for open manifolds.

I will conclude with some arithmetic applications of representation-stable homology with J. Ellenberg and B. Farb. For example, thinking of Conf_n as a scheme, the space of degree- n squarefree polynomials over \mathbb{F}_q is exactly $\text{Conf}_n(\mathbb{F}_q)$, and certain counting problems can be solved in terms of the multiplicities of irreducible representations in $H_i(\text{PConf}_n(\mathbb{C}); \mathbb{Q})$. (Received September 14, 2010)